Year 6

Pascal's Triangle Investigation

TRANSITION WORK

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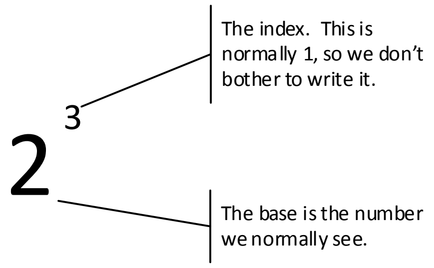
Indices

A number has two parts.



In this case, we have written the number 2 to the power of 3. This is sometimes called, "two cubed."

Two cubed means we do 2×2×2 which is 8.

Note that we *do not* do 2×3.

See if you can work out the value of the following sets of numbers:

1) $4^2 = 4 \times 4 =$
2) 5^2
3) 6^2
4) $2^4 = 2 \times 2 \times 2 \times 2 =$
5) 3^3
6) 4^3
7) 7^2
8) $3^2 \times 5^2 = 3 \times 3 \times 5 \times 5 =$
9) $2^3 \times 3 \times 4^2 = 2 \times 2 \times 2 \times 3 \times 4 \times 4 =$
10) $3^3 \times 5^2 \times 6 =$
11) $4^3 \times 5 =$
12) $2^6 \times 3^2 =$

You can check your answers with those at the back of the booklet.

Factors and Multiples

Factors are numbers that go into another number. They are always integers (which is a mathematical word for whole numbers). You can think of them as the times tables which contain the final number.

Four is a factor of 20 as 4 × 5 makes 20.

Five is a factor of 20 as 4 × 5 makes 20.

$$4 \times 5 = 20$$

Twenty is a multiple of four and five. It is also a multiple of 1, 2, 10 and 20.

So, if you were to ask, **"What are the factors of 20?"** the answer would be 1, 2, 4, 5, 10, 20. This is because 1 × 20 = 20, 2 × 10 = 20 and 4 × 5 = 20.

Alternatively, if you were to ask, **"What are the multiples of 4?"** then the answer would be 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, … and so on. That is because these numbers are in the four times table.

1) What are the multiples of 5 that are less than 100?
2) What are the multiples of 20 that are between 100 and 200?
3) What are the multiples of 6 that are between 30 and 60?
4) What are the factors of 24?
5) What are the factors of 50?
6) List the factors of 30 and then the factors of 54. Are there any factors that appear in both lists?

Factors that appear in two lists of factors are common to both lists. For this reason, we call them common factors.

Prime numbers

A prime number is any number that has exactly two factors. Prime numbers are really important in mathematics as they form the building blocks of the whole subject.

Numbers that appear as the answer in a times table question greater than one times something are not prime numbers.

In the exercise below, you have to go through each number crossing out all the numbers that appear in the two times table (with the exception of 1×2). Then you cross out all the numbers that appear in the three times table (with the exception of 1×3). Keep doing this up to the 12 times table. The numbers you should have left are the prime numbers. Remember though, 1 is not a prime number.

Eratosthenes Sieve

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104	105	106	107	108
109	110	111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130	131	132
133	134	135	136	137	138	139	140	141	142	143	144

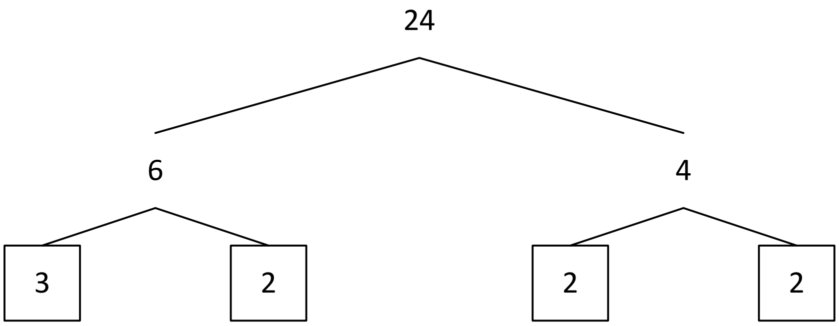
Once you have worked out the prime numbers, write them down in a list.

Prime Factors

As you have seen, we can split numbers into factors. For example, 6×4=24 so we can split 24 into factors of 6 and 4.

However, neither 6 nor 4 are prime numbers. These can be split further still: 3×2=6; 2×2=4.

So we can split the number 24 into prime factors:



You will notice that there is a box around the numbers 2 and 3. That is because they are prime numbers and so are prime factors of the number 24. You will notice that there are three 2s and one 3.

So the prime factors of 24 are $2^3 \times 3 = 2 \times 2 \times 2 \times 3 = 24$.

Some of you will know that you could split 24 in different ways, for example, 2×12 or 3×8.

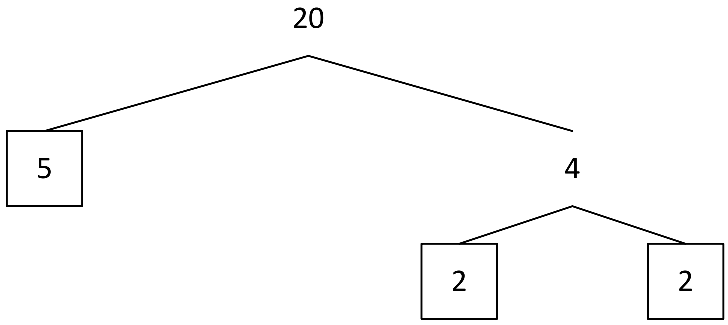
Draw a diagram like the one above, but this time, split 24 into 2×12 or 3×8. Then split each of those numbers up and then again until you get down to the prime factors. What do you notice?

How many different ways can you work out the prime factors of 36?

How many different ways can you work out the prime factors of 45?

Fundamental Theorem of Arithmetic

This is a posh way name for a rule in mathematics that says that every number greater than one can be made from the product of prime numbers. These prime numbers are called prime factors. The prime factors making up any number are unique.



Above is a diagram for working out the prime factors of 20. The prime factors of 20 are $2^2 \times 5 = 20$.

Use the method shown above to work out the prime factors of all the numbers between 20 and 40.

Write your answers like this:

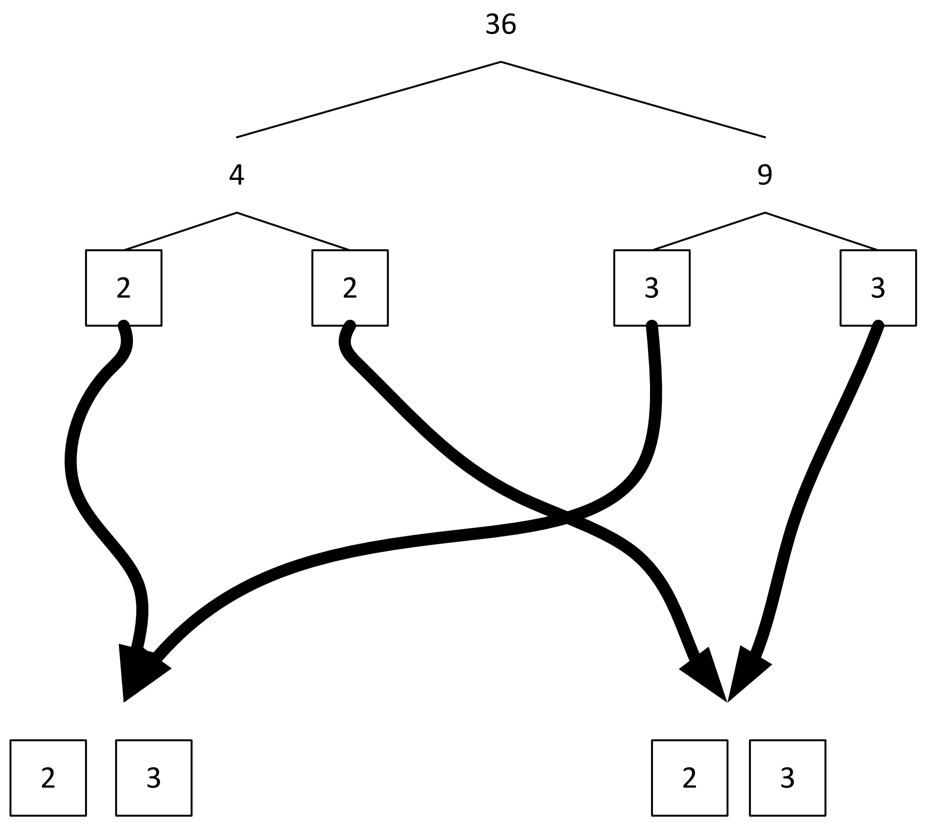
$20 = 2^2 \times 5$

$21 = 3 \times 7$ etc.

You will notice that 21 has only two factors. What other numbers only have two factors? What do we call these numbers?

Square Numbers

If we can put the prime factors of a number into two equal piles, we have a square number.

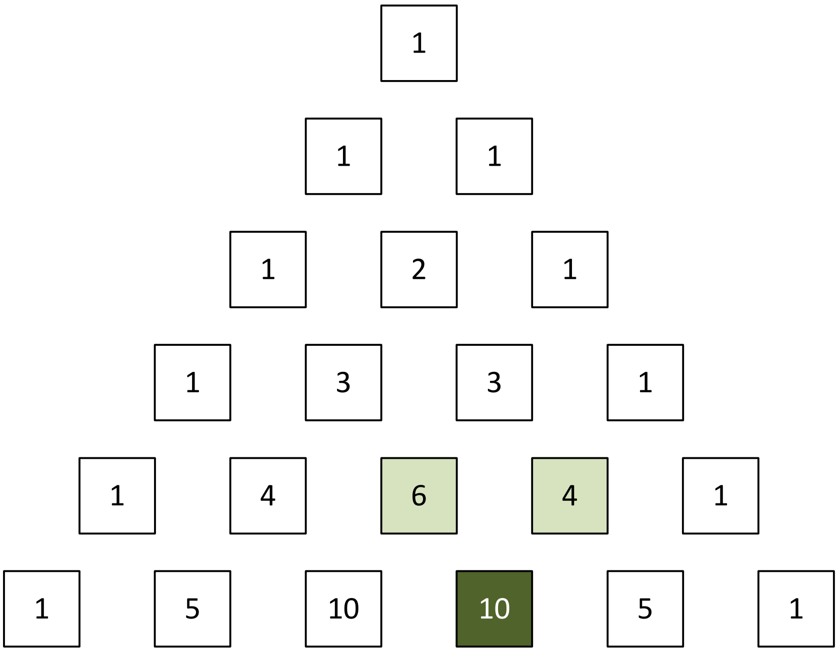


So in this case, 2 × 3 = 6. The square root of 36 is 6.

Use this method to find the square root of these numbers:

1. 225
2. 1024
3. 729
4. 5329
5. 6889

Pascal's Triangle



Above, is the start of a pattern of numbers called Pascal's Triangle. It is named after a French mathematician called Blaise Pascal as he was the first person in Europe to study it in any particular detail.

You will notice that the 6 and 4 are shaded in light green. The 10 beneath them is shaded in dark green. That is because each number is the sum of the two numbers above it.

Print off and cut out the numbers in Appendix A. See if you can use them to build up Pascal's Triangle. (There are some spares).

Pascal's Triangle (2)

The diagram below shows the names of each of the rows on Pascal's Triangle.

Row 0	1					
Row 1	1	1				
Row 2	1	2	1			
Row 3	1	3	3	1		
Row 4	1	4	6	4	1	
Row 5	1	5	10	10	5	1

Add up the numbers in each row. Do you notice anything about them? Can you spot a pattern in these numbers?

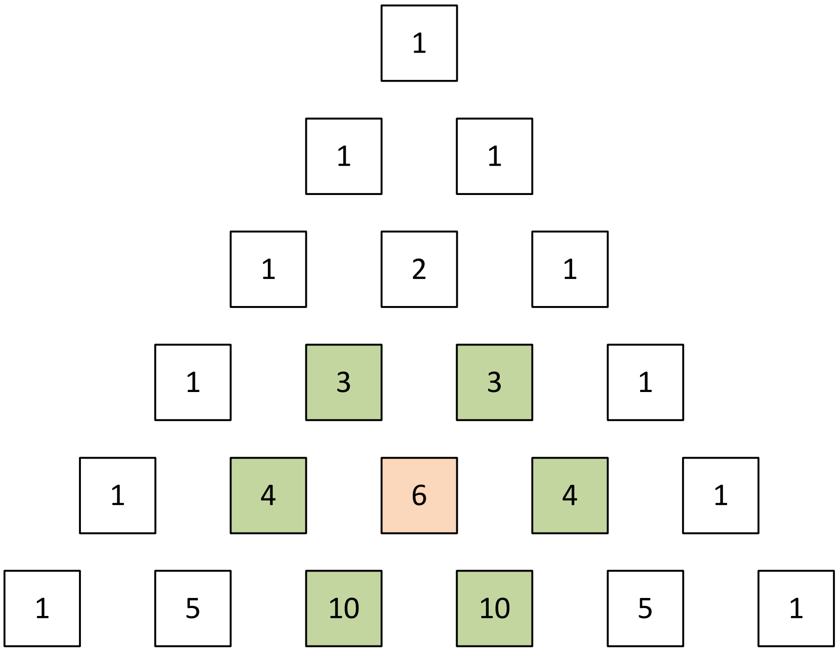
Can you use the pattern you spot to predict what row 10 will add up to?

What do you think the sum of row 15 would be?

What about row 25?

Pascal's Triangle (3)

Look at the triangle below. You will notice that one number is coloured in orange while the six numbers around it are coloured in green.



When you multiply numbers together, the answer you get is called the product.

If you find the product of those six numbers, you get an answer of 14,400. You can split this number into its factors (Remember that 3, 3, 4, 4, 10 and 10 will be factors of the number). If you continue to work this out, you can work out the prime factors of 14,400. In this case. You will get $2^6 \times 3^2 \times 5^2$.

You could split these prime factors into two equal piles giving you $2^3 \times 3 \times 5$ in each pile.

This means that 14,400 is a square number.

Using this method, see if you can find any other examples of square numbers in Pascal's Triangle.

Initially, you might find it more obvious to stick to the centre of the triangle, but try moving over to the edges as well to see if you can find any there.

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
1	2	3	3
4	4	6	5
5	10	10	6
6	15	15	7

20	15	6	21
21	7	7	35
35	8	8	28
28	56	56	70
9	9	36	36
84	84	126	126
10	45	120	210
10	45	120	210
252	11	11	55
55	165	165	330

462	462	330	165
21	1	7	2
35	1	8	1
28	56	55	70
9	20	21	36
2	3	4	5
6	7	8	9
10	11	12	210
252	84	120	55
55	165	165	330